LAST

Magnetic Reconnection in Weakly Collisional Plasmas: Paradox and Nature of Excited Modes

B. Coppi and T. Zhou
Massachusetts Institute of Technology, Cambridge, MA, 02139 USA

Modes producing magnetic reconnection in weakly collisional plasmas have been observed recently to have a phase velocity in the direction of the ion diamagnetic velocity. This challenges the so-called drift-tearing mode theory which predicted an opposite phase velocity direction. To solve the paradox, a two-fluid reconnection theory is formulated in which a "mode inductivity" is introduced to decouple the relevant plasma motion from the magnetic field lines. This may be viewed as representing the electromagnetic coupling of the current channels inside the reconnection layer with those outside it. The relevant theory leads to finding a mode having a phase velocity slightly lower than the ion diamagnetic velocity and in the same direction. The reconnection layer thickness is proportional to the mode inductivity and the mode growth rate is associated with the ion momentum diffusion. A less preferable alternative is a resistive reconnecting mode, with a phase velocity in the ion diamagnetic velocity direction, requiring a large anomalous plasma resistivity for the validity of the adopted two-fluid theory. In this case the reconnection layer width would be proportional to the plasmas resistivity and the growth rate to the cube of it.

PACS numbers: 52.30.Ex, 52.35.Py, 52.35.Vd
Keywords: magnetic reconnection, mode inductivity

An important contribution towards understanding the nature of the modes that produce magnetic reconnection in weakly collisional plasmas is given by the experimental observation of their phase velocities, in particular of the direction of these relative to the particle diamagnetic velocities in well confined plasmas. In fact, the first theory of modes producing magnetic reconnection in weakly collisional regimes had led to identifying the so-called drift-tearing modes [1, 2] that have a phase velocity in the direction of the electron diamagnetic velocity. Thus, for a relatively long time, these modes have been considered as responsible for the reconnection leading to the formation of the observed magnetic islands. In the meantime a theoretical problem that has emerged, starting from the theory of these modes in collisionless regimes [3], is that the presence of an electron temperature gradient prevents their instability. Consequently the additional effects of other modes [4] had to be introduced in order to justify their excitation.

More recently however, a systematical investigation [5] on the phase velocities of modes producing magnetic islands in plasmas produced by different machines, in particular the JET and the Frascati Torus devices with varying degrees of collisionality, has revealed that the direction of these velocities is that of the ion diamagnetic velocity. The exploration [6] of high-beta discharges on the JET machine indicates that an $n = 1$ instability can develop a tearing topology with the reconnection layer centered on the $q = 2$ surface. An additional insight is given by the direction of the "spontaneous rotation " of the background plasma that is recognized [7, 8] to be associated with these modes. Then these observations have led us to propose a two-fluid theoretical model that introduces a "mode inductivity" [9] in the longitudinal electron momentum conservation equation considering the plasma resistivity to be negligible. This mode inductivity may represent the electromagnetic coupling of the current channels inside the reconnection layer with those outside it, and may be viewed as a "strongly enhanced electron mass". According to the dispersion relation obtained by matching the solution inside the reconnection layer with that of the outer ideal MHD region, the mode phase velocity is slightly below the ion diamagnetic velocity and is in the same direction. The growth rate of this mode is associated to the transverse ion momentum diffusion.

We point out that introducing a considerable resistivity a reconnecting mode with a phase velocity in the ion diamagnetic velocity direction can be found but with a thickness of the reconnection layer proportional to the plasma resistivity. Thus a large resistivity enhancement over its classical expression is required in order to ensure that the layer width is larger than the ion gyro-radius for the validity of the adopted two-fluid theory.

An important feature of the reconnecting modes that has been analyzed first in Ref. [8] is that they extract angular momentum from the background plasma as they grow, according to the theory of "spontaneous rotation" in axi-symmetric plasmas given in Refs. [7, 8]. Consistently with this, the rotation is in the direction of the electron diamagnetic velocity (counter-current direction) since the phase velocity of the reconnecting mode is in the direction of ion diamagnetic velocity. This appears to be confirmed by the experimental results reported in Ref. [10]. The rotation profile of the background plasma has a characteristic radial distribution that can be viewed as the superposition of a monotonically decreasing profile on the radial variable and a local "bump" that is proposed [8] to be associated with the considered reconnecting modes.

Basic Equations.–We take the guiding center point of view and consider at first the collisionless longitudinal (to the magnetic field) electron momentum conservation equation as consisting of the terms indicated in the following

$$0 \simeq -\nabla_{\parallel}\hat{p}_e - \frac{\widehat{B}_x}{B}\frac{dp_e}{dx} - en\widehat{E}_{\parallel} \quad (1)$$

where $\widehat{E} \simeq -\nabla\hat{\phi} - e_z\partial\hat{A}_z/\partial t/c$ and $\widehat{B}_x \simeq ik_y\hat{A}_z$. Therefore Eq. (1) that can be appropriate for collisionless regimes yields

$$0 \simeq -ik_{\parallel}\hat{p}_e - ik_y\frac{\hat{A}_z}{B}\frac{dp_e}{dx} - ien\left(\frac{\omega}{c}\hat{A}_z - k_{\parallel}\hat{\phi}\right) \quad (2)$$

where $k_{\parallel} = (k\cdot B)/B$ and, near $x = x_0$, $k_{\parallel} = (k\cdot B')/B \cdot (x - x_0) \simeq [k_z + k_y(dB_y/dx)/B](x - x_0)$. Consequently

$$\frac{1}{c}\left(\omega - \omega^p_{*e}\right)\hat{A}_z \simeq k_{\parallel}\left(\hat{\phi} - \frac{1}{en}\hat{p}_e\right) \quad (3)$$

where $\omega^p_{*e} \equiv -k_y[c/(enB)]\,dp_e/dx$ and $\hat{A}_z = 0$ and $\widehat{B}_x = 0$ for $k_{\parallel} = 0$. We note that in high temperature regimes, where the longitudinal electron thermal conductivity is relatively large, $\left|\widehat{T}_e/T_e\right| \ll |\hat{n}_e/n_e|$. Therefore $\hat{p}_e \simeq T_e\hat{n}_e$ and $-i\omega\hat{n}_e + \hat{v}_{Ex}(dn/dx) + ik_{\parallel}\hat{u}_{e\parallel} \simeq 0$ where $\hat{v}_{Ex} = -ick_y\hat{\phi}/B$. Thus

$$\hat{p}_e \simeq nT_e\left(\frac{\omega_{*e}}{\omega}\frac{e\hat{\phi}}{T_e} + \frac{k_{\parallel}}{\omega}\hat{u}_{e\parallel}\right) \quad (4)$$

where $\omega_{*e} \equiv -k_y[cT_e/(enB)]\,dn/dx$ and Eq. (3) becomes

$$\frac{1}{c}\left(\omega - \omega^p_{*e}\right)\hat{A}_z \simeq k_{\parallel}\left[\hat{\phi}\left(1 - \frac{\omega_{*e}}{\omega}\right) - \frac{T_e}{e}\frac{k_{\parallel}}{\omega}\hat{u}_{e\parallel}\right]. \quad (5)$$

In order that reconnection take place we introduce an "inductive" term in the longitudinal momentum conservation equation that may represent the effects of an electromagnetic coupling to other ongoing current carrying processes. Thus we have, instead of Eq. (1)

$$0 \simeq -\nabla_{\parallel}\hat{p}_e - \frac{\widehat{B}_x}{B}\frac{dp_e}{dx} - en\left[\widehat{E}_{\parallel} + i\omega\mathcal{L}\hat{J}_{\parallel}\right] \quad (6)$$

where $\mathcal{L} \equiv (4\pi/c^2)\,S_L$ is the relevant mode inductivity and $\hat{J}_{\parallel} \simeq -en\hat{u}_{e\parallel}$.

Now we note that

$$k_{\parallel}\hat{p}_e + \omega en\left(\frac{4\pi}{c^2}S_L\right)\hat{J}_{\parallel} \simeq k_{\parallel}nT_e\left[\frac{\omega_{*e}}{\omega}\frac{e\hat{\phi}}{T_e} + \frac{k_{\parallel}}{\omega}\hat{u}_{e\parallel}\right] - \omega n\frac{4\pi ne^2}{c^2}S_L\hat{u}_{e\parallel} \quad (7)$$

and we have to compare $k_{\parallel}^2T_e/\omega^2$ to $4\pi ne^2S_L/c^2$ or $k_{\parallel}^2v_s^2/\omega^2$ to $\omega_{pi}^2S_L/c^2$ where $v_s^2 \equiv T_e/m_i$ and $\omega_{pi}^2 \equiv 4\pi ne^2/m_i$, that is $k_y^2\left(B_y'^2/B^2\right)\left(\delta^2v_s^2/\omega^2\right)$ to $\omega_{pi}^2S_L/c^2$. As we shall see $\delta^2 \sim [S_L/(r_J I)]^2$ where $r_J \sim |B_y/B_y'|$ and $I^2 > 1$. Therefore we compare $\left(c^2/\omega_{pi}^2\right)\left(k_y^2/\omega^2\right)\left(B_y^2/B^2\right)\left[\left(S_Lv_s^2\right)/\left(r_J^4I^2\right)\right]$ to 1 or

$$2\frac{c^2}{\omega_{pi}^2\rho_i^2}\frac{T_e}{T_i}\frac{B_y^2}{B^2}\frac{S_Lr_{pi}^2}{r_J^4I^2} \quad \text{to} \quad 1 \quad (8)$$

for $\omega \sim \omega_{di}$ and $r_{pi} \equiv |p_i/(dp_i/x)|$. Thus we may proceed considering the first term in Eq. (8) to be well below unity.

Moreover, if we define $\hat{\xi}_x \equiv i\hat{v}_{Ex}/\omega$ the x-component of the equation $c\nabla\times\widehat{E} = -\partial\widehat{B}/\partial t$ takes the form

$$ck_y\widehat{E}_{\parallel} \simeq \omega\left(\widehat{B}_x - ik\cdot B\hat{\xi}_x\right). \quad (9)$$

Finally, using Eq. (9) and noticing that $\hat{J}_{\parallel} \simeq i[c/(4\pi k_y)]\,\partial^2\widehat{B}_x/dx^2$ and that $\nabla_{\parallel}\hat{p}_e \simeq ik_{\parallel}T_e\hat{n}_e \simeq -ik_{\parallel}T_e(dn/dx)\hat{\xi}_x$ that is valid in high temperature regimes, we obtain from Eq. (6)

$$\frac{\omega - \omega^p_{*e}}{\omega - \omega_{*e}}\widetilde{B}_x \simeq i(k\cdot B)\tilde{\xi}_x + \frac{S_L\omega}{\omega - \omega_{*e}}\frac{d^2\widetilde{B}_x}{dx^2}. \quad (10)$$

The other equation to be coupled with Eq. (10) is the quasi-neutrality condition

$$\nabla\cdot\hat{J}_{\perp} + \nabla\cdot\widehat{bb\cdot J} = 0 \quad (11)$$

where $b \equiv B/B$ and we adopt the guiding center point of view. Therefore

$$\hat{J}_{\perp} \simeq en\left(\hat{v}_{pi} + \hat{v}_{FLR}\right) \quad (12)$$

where $\hat{v}_{pi} = -i(\omega/\Omega_{ci})\left(\widehat{E}_{\perp}/B\right)c$ is the ion polarization drift and $\hat{v}_{FLR} = i(\omega_{di}/\Omega_{ci})\left(\widehat{E}_{\perp}/B\right)c$ is the finite Larmor radius drift with $\omega_{di} = k_yv_{di}$ and $v_{di} \equiv [c/(enB)]\,dp_i/dx$ being the ion diamagnetic velocity. Consequently $\nabla\cdot\hat{J}_{\perp} \simeq i[en/(\Omega_{ci}k_y)]\,\omega(\omega - \omega_{di})\,\partial^2\hat{\xi}_x/\partial x^2$.

Moreover, $\nabla\cdot\widehat{bb\cdot J} = \left(\widehat{B}_x/B\right)(dJ_{\parallel}/dx) + ik_{\parallel}\hat{J}_{\parallel}$ where $J_{\parallel} = [c/(4\pi)]\,B_y'$, $B_y' \equiv dB_y/dx$ and $\hat{J}_{\parallel} \simeq i[c/(4\pi k_y)]\left[d^2\widehat{B}_x/dx^2\right]$.

Then Eq. (11) for the inner region where $\left|d^2/dx^2\right| > k^2$ can be rewritten as

$$-\rho\omega(\omega - \omega_{di})\frac{d^2\tilde{\xi}_x}{dx^2} \simeq i\frac{k\cdot B}{4\pi}\frac{d^2\widetilde{B}_x}{dx^2} - i\frac{k_yB_y''}{4\pi}\widetilde{B}_x \quad (13)$$

where $B_y'' = d^2B_y/dx^2$.

Matching Condition and Reconnection Layer.–The matching condition between the solution in the inner region and that in the outer region involves the parameter r_J defined as

$$\frac{1}{r_J} = \frac{1}{\widetilde{B}_{x0}}\left\{\left.\frac{d\widetilde{B}_x}{dx}\right|_{x=x_{0+}} - \left.\frac{d\widetilde{B}_x}{dx}\right|_{x=x_{0-}}\right\}, \quad (14)$$

where $\widetilde{B}_x$ is found by solving the equation

$$0 \simeq (k \cdot B)\left(\frac{d^2\widetilde{B}_x}{dx^2} - k^2\tilde{B}_x\right) - k_y\frac{d^2B_y}{dx^2}\widetilde{B}_x \quad (15)$$

in the outer region ($|x - x_0| \gg \delta$). We note that Eq. (15) is derived from

$$\nabla_{\parallel}\hat{J}_{\parallel} + \frac{\widehat{B}_x}{B}\frac{dJ_{\parallel}}{dx} \simeq 0. \quad (16)$$

It is clear that a positive r_J corresponds to relatively low values of k^2. Then the matching condition is

$$\frac{1}{r_J} = \frac{1}{\widetilde{B}_{x0}\delta}\int_{-\infty}^{+\infty}\left.\frac{d^2\widetilde{B}_x}{d\bar{x}^2}\right)_{\text{in}} d\bar{x} \quad (17)$$

where $\bar{x} \equiv (x - x_0)/\delta$ and $d^2\widetilde{B}_x/dx^2)_{\text{in}}$ is evaluated as a function of $\bar{x}$ in the inner region. In particular, referring to Eq. (10) we have

$$\left.\frac{1}{\widetilde{B}_{x0}}\frac{d^2\widetilde{B}_x}{d\bar{x}^2}\right)_{\text{in}} \simeq \frac{\omega - \omega_{*e}^p}{S_L\,\omega}\left\{1 - i\,(k\cdot B)\frac{\tilde{\xi}_x}{\widetilde{B}_{x0}}\frac{\omega - \omega_{*e}}{\omega - \omega_{*e}^p}\right\}$$

$$= \frac{\omega - \omega_{*e}^p}{S_L\,\omega}\{1 - \bar{x}Y\} \quad (18)$$

where $\omega_{*e}^p \equiv -k_y\left[c/(enB)\right]dp_e/dx$, $\omega_{*e} \equiv -k_y dn/dx \cdot \left[cT_e/(enB)\right]$ and

$$Y \equiv \frac{i\,(k\cdot B')_0}{\widetilde{B}_{x0}}\left(\tilde{\xi}_x\delta\right)\frac{\omega - \omega_{*e}}{\omega - \omega_{*e}^p}. \quad (19)$$

Consequently Eq. (17) becomes

$$\frac{1}{r_J} = \frac{\delta}{S_L}\frac{\omega - \omega_{*e}^p}{\omega}\int_{-\infty}^{+\infty} d\bar{x}\left[1 - \bar{x}Y(\bar{x})\right]. \quad (20)$$

As we shall show the integral in Eq. (20) is independent of ω and is positive. That is

$$I \equiv \int_{-\infty}^{+\infty} d\bar{x}\left[1 - \bar{x}Y(\bar{x})\right] > 0 \quad (21)$$

and

$$\delta = \frac{S_L}{r_J I}\frac{\omega}{\omega - \omega_{*e}^p}. \quad (22)$$

Since $\delta \sim S_L/r_J$, for the validity of the adopted two-fluid theory we require that

$$\delta > \rho_i, \quad (23)$$

where $\rho_i \equiv v_{thi}/\Omega_{ci}$ is the ion gyro-radius. Thus we need

$$S_L > \rho_i r_J. \quad (24)$$

As we shall verify the relevant dispersion relation gives, for the mode of interest, $\omega = \omega_{di} - \delta\omega$ with $0 < \text{Re}(\delta\omega)/\omega_{di} \ll 1$. Therefore

$$\delta \simeq \frac{S_L}{r_J I}\frac{dp_i/dx}{d(p_i + p_e)/dx}. \quad (25)$$

At this point we may propose a magnitude and an expression for S_L considering its relationship to δ as given by Eq. (25). Thus we relate δ to c/ω_{pi}, "the ion inertia skin depth", and in particular, since $\omega \simeq \omega_{di}$ we take

$$\delta \simeq \alpha_i^p\frac{c}{\omega_{pi}}\frac{p_i + p_e}{|d(p_i + p_e)/dx|}\frac{1}{r_J} \quad (26)$$

where α_i^P is a finite numerical coefficient. Then we obtain

$$S_L \simeq \alpha_i^p\frac{c}{\omega_{pi}}\frac{I(p_i + p_e)}{|dp_i/dx|}. \quad (27)$$

It is evident that the validity of the adopted linearized theoretical model is limited to the size of the produced magnetic islands remaining smaller than δ, that is

$$\left|\frac{r_J\widetilde{B}_x}{B_y'}\right|^{1/2} < \delta. \quad (28)$$

Finally, we argue that the resulting unstable mode represents the beginning of a non-linear process involving magnetic reconnection that should "remember" the original mode phase velocity.

Dispersion Relation.–For the sake of simplicity (in particular, in order to deal with eigen-functions with one parity), we consider d^2B_y/dx^2 at $x = x_0$ to be negligible. Then referring to Eqs. (10) and (13) we have

$$-\omega(\omega - \omega_{di})\frac{d^2\tilde{\xi}_x}{dx^2} \simeq i\frac{(k\cdot B)}{4\pi\rho}\frac{(\omega - \omega_{*e}^p)}{\omega S_L}\widetilde{B}_{x0} \quad (29)$$

$$\cdot\left\{1 - i\,(k\cdot B)\frac{\tilde{\xi}_x}{\widetilde{B}_{x0}}\frac{\omega - \omega_{*e}}{\omega - \omega_{*e}^p}\right\}$$

and we note that when $0 < \omega/\omega_{di} < 1$ the solution of Eq. (29) is a decaying function of $\bar{x}$ for $\bar{x}^2 \to \infty$. In particular, by introducing the definition of Y given in Eq. (19), Eq. (29) becomes

$$\frac{d^2Y}{d\bar{x}^2} = \bar{x}\left(\bar{x}Y - 1\right) \quad (30)$$

if we take

$$\delta^4 = \frac{S_L}{k^2}\frac{\omega^2}{\omega_{\mathcal{H}}^2}\frac{\omega_{di} - \omega}{\omega - \omega_{*e}} \quad (31)$$

where $\omega_{\mathcal{H}}^2 \equiv B_y'^2/(4\pi\rho)$. Thus we arrive at the dispersion relation

$$\frac{\omega_{di} - \omega}{\omega - \omega_{*e}}\frac{(\omega - \omega_{*e}^p)^4}{\omega^2\omega_{\mathcal{H}}^2} \simeq S_L^3\frac{k^2}{(r_J I)^4}. \quad (32)$$

The relevant root is $\omega \simeq (1 - \delta\bar{\omega})\,\omega_{di}$ where

$$\delta\bar{\omega} \simeq \left(1 - \frac{\omega_{*e}}{\omega_{di}}\right)\frac{\omega_{di}^2\omega_{\mathcal{H}}^2}{(\omega_{di} - \omega_{*e}^p)^4}\frac{k^2S_L^3}{(r_J I)^4} \sim \frac{\omega_{\mathcal{H}}^2}{\omega_{di}^2}\frac{k^2S_L^3}{r_J^4}, \quad (33)$$

for $\omega_{di} \sim \omega_{*e}^{p} \sim \omega_{*e}$. Clearly, in this case, Eq. (29) reduces to

$$\omega_{di}^2 \delta\bar{\omega} \frac{d^2\tilde{\xi}_x}{dx^2} \simeq \frac{\omega_{\mathcal{H}}^2}{S_J} \left\{ k^2 (x - x_0)^2 \left(1 - \frac{\omega_{*e}}{\omega_{di}}\right) \tilde{\xi}_x + k (x - x_0) \left(1 - \frac{\omega_{*e}^{p}}{\omega_{di}}\right) \left(i \frac{\widetilde{B}_{x0}}{B_y'}\right) \right\} \quad (34)$$

and we note that $\delta\bar{\omega}$ is independent of k. Moreover, the condition $\delta > \rho_i$ implies that

$$\delta\bar{\omega} > \rho_i / r_J . \quad (35)$$

An approximate solution of Eq. (30) that has the appropriate asymptotic limits for both $|\bar{x}| \gg 1$ and $|\bar{x}| \ll 1$ is

$$Y(\bar{x}) \simeq \frac{\bar{x}}{\sqrt{6} + \bar{x}^2}, \quad (36)$$

but is not sufficiently accurate for the evaluation of the quantity I that would be $I \simeq \sqrt[4]{6}\,\pi$. In fact P. Montag has shown that the appropriate value of I is about 2 and we note that the quantity I enters as a considerable reduction factor in the expression for $\delta\bar{\omega}$.

Importance of Nuclei Momentum Transfer.–Although the geometric characteristics of the inductive modes identified by the previous analysis comply with the requirements for the excitation of bound and non-convective [11] normal modes, a growth rate remains to be found. For this we consider the effects of nuclei collisions or of an effective transverse (to the magnetic field) viscosity. As we shall show, a growth rate will be associated with these effects.

The most rudimentary way to illustrate this point is to introduce a rate of momentum dissipation ν_μ in the total momentum conservation equation. Thus, instead of Eq. (13) we obtain

$$-(\omega + i\nu_\mu)(\omega - \omega_{di}) \frac{d^2\tilde{\xi}_x}{dx^2} \simeq i \frac{k \cdot B}{4\pi\rho} \frac{d^2\widetilde{B}_x}{dx^2} \quad (37)$$

$$\simeq i \frac{(k \cdot B)}{4\pi\rho} \frac{(\omega - \omega_{*e}^{p})}{\omega S_L} \widetilde{B}_{x0} \left\{ 1 - i (k \cdot B) \frac{\tilde{\xi}_x}{\widetilde{B}_{x0}} \frac{\omega - \omega_{*e}}{\omega - \omega_{*e}^{p}} \right\}. \quad (38)$$

Then we proceed with the same analysis that has led to Eq. (32) and obtain

$$\omega \simeq (1 - \delta\bar{\omega}) \omega_{di} + i\gamma, \quad (39)$$

where $\delta\bar{\omega}$ is positive and given by Eq. (33) and

$$\gamma = \nu_\mu \delta\bar{\omega}. \quad (40)$$

Clearly, this shows that the mode is weakly unstable. In particular, we see that both $\delta\bar{\omega}$ and γ are independent of k_y.

At this point we may consider instead of ν_μ a viscous operator such as that due to ion-ion collisions and obtain, instead of Eq. (38)

$$-(\omega - \omega_{di}) \left(\omega - iD_\mu \frac{d^2}{dx^2}\right) \frac{d^2\tilde{\xi}_x}{dx^2} \simeq i \frac{(k \cdot B)}{4\pi\rho} \frac{d^2\widetilde{B}_x}{dx^2} \quad (41)$$

where D_μ is the transverse "viscous" diffusion coefficient. Consequently, instead of Eq. (29), we consider

$$-(\omega - \omega_{di}) \left(\omega - iD_\mu \frac{d^2}{dx^2}\right) \frac{d^2\tilde{\xi}_x}{dx^2} \quad (42)$$

$$\simeq i \frac{(k \cdot B)}{4\pi\rho} \frac{(\omega - \omega_{*e}^{p})}{\omega S_L} \widetilde{B}_{x0} \left\{ 1 - i (k \cdot B) \frac{\tilde{\xi}_x}{\widetilde{B}_{x0}} \frac{\omega - \omega_{*e}}{\omega - \omega_{*e}^{p}} \right\}.$$

Using the variables introduced previously, this can be rewritten as

$$\left(1 - i \frac{D_\mu}{\omega\delta^2} \frac{d^2}{d\bar{x}^2}\right) \frac{d^2 Y}{d\bar{x}^2} \simeq -\bar{x}(1 - \bar{x}Y), \quad (43)$$

which has the solution $Y \simeq Y_0 + i \left[D_\mu / (\omega\delta^2)\right] Y_\mu$ assuming that $\left|D_\mu / (\omega\delta^2)\right| \ll 1$. Here Y_0 is the solution of Eq. (30) and Y_μ is determined by the equation

$$Y_\mu'' - \bar{x}^2 Y_\mu = Y_0^{(4)}. \quad (44)$$

Therefore, the dispersion relation Eq. (20) can be written as

$$\frac{1}{r_J} = \frac{\delta(\omega)}{S_L} \frac{\omega - \omega_{*e}^{p}}{\omega} \left[I_1 + i \frac{D_\mu}{\omega\delta^2} I_2\right] \quad (45)$$

where

$$I_1 \equiv -\int_{-\infty}^{\infty} d\bar{x} \frac{Y_0''(\bar{x})}{\bar{x}}, \quad I_2 \equiv -\int_{-\infty}^{\infty} d\bar{x}\bar{x} Y_\mu > 0. \quad (46)$$

With the definition of δ given by Eq. (31), the dispersion relation that can be obtained is similar to Eq. (32) except that I is replaced by $I\left\{1 + i4(I_2/I)\left[D_\mu / (\omega\delta^2)\right]\right\}$. If we take $\omega \simeq \omega_0 + i\gamma$ where $\omega_0 \simeq (1 - \delta\bar{\omega})\omega_{di}$, we obtain the growth rate

$$\gamma \simeq 4 \frac{I_2}{I} \frac{D_\mu}{\delta^2} (\delta\bar{\omega}). \quad (47)$$

Clearly, the growth rate is independent the sign of k.

A Resistive Mode?–A mode that involves reconnection and has a phase velocity in the direction of the ion diamagnetic velocity can be found, although with strong limitations. These are connected with the fact that the resistivity introduced to break the "frozen-in" condition has to be relatively large. That is, we adopt the following from of the longitudinal electron momentum balance equation

$$0 \simeq -\nabla_{\shortparallel} \hat{p}_e - \frac{\widehat{B}_x}{B} \frac{dp_e}{dx} - en \left(\widehat{E}_{\shortparallel} - \eta_{\shortparallel} \hat{J}_{\shortparallel}\right). \quad (48)$$

where $\eta_{\shortparallel} \equiv 4\pi D_m / c^2$ and D_m is the (resistive) magnetic field diffusion coefficient.

We proceed to obtain the radial differential equation that is valid in the inner reconnection layer and replaces Eq. (29). This is

$$\frac{\omega-\omega_{*e}^{p}}{\omega-\omega_{*e}}\widetilde{B}_x \simeq i\,(k\cdot B)\,\tilde{\xi}_x + i\frac{D_m}{\omega-\omega_{*e}}\frac{d^2\widetilde{B}_x}{dx^2}. \quad (49)$$

and is to be coupled to Eq. (13) inside the reconnection layer.

Then combining Eqs. (13) and (49) and taking $\widetilde{B}_x \simeq \widetilde{B}_{x0}$ we obtain

$$-\omega\,(\omega-\omega_{di})\frac{d^2\tilde{\xi}_x}{dx^2} \simeq \frac{(k\cdot B)}{4\pi\rho}\frac{(\omega-\omega_{*e}^{p})}{D_m}\widetilde{B}_{x0} \cdot \left[1 - i\,(k\cdot B)\frac{\tilde{\xi}_x}{\widetilde{B}_{x0}}\frac{\omega-\omega_{*e}}{\omega-\omega_{*e}^{p}}\right], \quad (50)$$

which can be rewritten as

$$\frac{d^2Y}{d\bar{x}^2} \simeq -\bar{x}\,(1-\bar{x}Y) \quad (51)$$

once we define the reconnection layer width δ_r as

$$\delta_r^4 = i\frac{D_m}{k_y^2}\frac{\omega}{\omega_{\mathcal{H}}^2}\frac{\omega_{di}-\omega}{\omega-\omega_{*e}}, \quad (52)$$

$\bar{x} \equiv (x-x_0)/\delta_r$ and Y as in Eq. (19) with δ being replaced by δ_r.

As we did previously, when we match the solution in the inner layer with that in the outer region, we obtain

$$\delta_r \simeq \frac{i}{I}\frac{D_m}{r_J\,(\omega-\omega_{*e}^{p})} \quad (53)$$

that is analogous to Eq. (22). Finally, Eqs. (52) and (53) lead to the dispersion relation

$$\omega\,(\omega-\omega_{di})\frac{(\omega-\omega_{*e}^{p})^4}{\omega-\omega_{*e}} = iD_m^3\omega_{\mathcal{H}}^2\frac{k^2}{(r_J I)^4} \quad (54)$$

that is similar to the dispersion relation (9.3) in Ref. [8] and was derived originally in Ref. [1], where the drift-tearing mode was identified for the first time.

In this dispersion relation, if we take $\omega \simeq \omega_{di} + i\gamma$ for $|\gamma| \ll |\omega_{di}|$ that corresponds to the mode phase velocity being in the direction of the ion diamagnetic velocity, we have the extremely small growth rate

$$\gamma \simeq \frac{(\omega_{di}-\omega_{*e})\,\omega_{\mathcal{H}}^2}{\omega_{di}\,(\omega_{di}-\omega_{*e}^{p})^4}\frac{k^2}{(r_J I)^4}D_m^3 > 0. \quad (55)$$

This feature, $\gamma \propto D_m^3$, is the reason why this root of Eq. (54) was not considered previously.

A good feature is that δ_r^4 is positive. Then the radial structure of the modes does not have internal oscillations unlike the case of the drift tearing mode with $\omega \simeq \omega_{*e}^{p}$. On the other hand Eq. (52) yields

$$\delta_r^4 \simeq \left[\frac{D_m}{r_J I\,|\omega_{di}-\omega_{*e}^{p}|}\right]^4. \quad (56)$$

Consequently the condition that δ_r exceeds the ion gyro-radius ρ_i, as is necessary for the validity of the adopted two-fluid description, is

$$\frac{D_m r_{pi}}{Ik_\perp D_B^i r_J\,(1-\omega_{*e}^{p}/\omega_{di})} > \rho_i \quad (57)$$

that is

$$D_m > D_B^i\frac{\rho_i}{r_{pi}}\,(k_\perp r_J I) \quad (58)$$

where $D_B^i \equiv cT_i/(eB)$, $r_{pi} \equiv |p_i/(dp_i/dx)|$ and $D_B^i\rho_i/r_{pi}$ is of the order of the so-called Gyro-Bohm diffusion coefficient [12]. Therefore a relatively large anomalous resistivity would be required for the validity of this option and the alternative of introducing a finite mode inductivity to explain the observed phase velocity of the modes is considered as preferable.

Clearly, the evolution of the considered mode into its nonlinear phase will have to be obtained by numerical means.

It is a real pleasure to thank P. Buratti and G. Pucella for the exchange of ideas maintained on the subject of this paper and P. Montag for his evaluation of the integral in Eq. (21). This work was sponsored by the U. S. Department of Energy.

---

[1] B. Coppi, Phys. Fluids **8**, (1965) 2273.

[2] G. Ara, B. Basu, B. Coppi, G. Laval, M. Rosenbluth and B. Waddell, Ann. of Phys. **112**, (1978) 443.

[3] B. Coppi, J. Mark, L. Sugiyama and G. Bertin, Phys. Rev. Lett. **42**, (1979) 1058.

[4] B. Coppi, *Collective Phenomena in Macroscopic Systems* [Publ. World Scientific (2007)], P. 59.

[5] P. Buratti, B. Coppi, G. Pucella and T. Zhou, Bull. Amer. Phys. Soc. **58**, (2013) 142.

[6] P. Buratti, M. Baruzzo, R. Buttery, C. Challis *et al.*, Nucl. Fusion **52**, (2012) 023006.

[7] B. Coppi, 18th IAEA Fusion Energy Conf. **THP 1/17** (2000); Nucl. Fusion **42**, (2002) 1.

[8] B. Coppi and T. Zhou, submitted to Nuclear Fusion, (2013).

[9] B. Coppi, Bull. Amer. Phys. Soc. **45**, (2000) 366.

[10] O. Sauter, B. Duval, L. Federspiel, F. Felici *et al.*, 23rd IAEA Fusion Energy Conf. **EXS/P2-17** (2010).

[11] B. Coppi, G. Laval, R. Pellat and M. Rosenbluth, Nuclear Fusion **6**, (1966) 261.

[12] B. Coppi, M. Rosenbluth and R. Sagdeev, Phys Fluids **10**, (1967) 582.